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基于三角函数的犹豫模糊新距离测度

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摘　要：距离测度主要用于度量不同数据之间的距离，结合三角函数的距离测度也被成功地应用于犹豫模糊集上。本文提出了基于正弦、余弦和正切函数的犹豫模糊距离测度以及含偏好的距离测度并验证了其优良性质；最后，提出了2种犹豫模糊多属性决策方法，并通过能源政策选择的案例验证了所提方法的有效性和实用性。

关键词：犹豫模糊集；三角函数；距离测度；多属性决策

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New Distance Measure of Hesitant Fuzzy Set Based on Trigonometric Function

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Abstract: Distance measure was mainly used to measure the distance between different data. The distance measure combined with trigonometric function has been successfully applied in the hesitant fuzzy set. In this paper,we proposed new distance measures based on the hesitant fuzzy set of sine, cosine and tangent functions, and distance measure with preference information. Further, the properties of the proposed distance measure were discussed. Finally, two multi-attribute decision making methods based on hesitant fuzzy sets were proposed. The effectiveness and practicality of the proposed methods were demonstrated by the example of energy policy selection.

Keywords: hesitant fuzzy set; trigonometric function; distance measure; multi-attribute decision-making

模糊集（FS）是处理复杂性和不确定性问题的有力工具之一，于1963年被美国控制论专家Zadeh[1]提出。文献[2-3]考虑到隶属度、非隶属度和犹豫度3个因素之间的影响，提出了直觉模糊集（IFS）、区间直觉模糊集（IVIFS）等拓展形式。在分析问题时，决策者会对一个元素属于某个集合的可能程度犹豫不决，或者决策者对同一数据在不同时期的评估值可能发生改变，由此，文献[4]提出了犹豫模糊集（HFS），允许隶属度在单位区间[0,1]上取一组可能值。

测度理论是解决决策问题的重要工具，有关犹豫模糊集的距离测度和相似性测度的研究已经取得了较多成果。文献[5]提出了犹豫Hamming距离和犹豫Hausdorff距离。文献[6]考虑到决策者在评估犹豫模糊数(HFN)的隶属度时的犹豫程度，引入了包含犹豫程度的距离测度。文献[7]为解决隶属度的偏差问题，定义了距离度量时每种补齐方案的犹豫度的直觉犹豫模糊集的距离测度。但上述距离测度受不同的犹豫模糊数所含元素个数等长的限制，这可能会导致结果与真实情况存在差异。针对这一问题，文献[8]提出了基于对

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偶犹豫模糊集的距离测度和熵测度,文献[9]提出了基于犹豫模糊集的不等长序列识别方法,有效地解决了犹豫模糊数的元素个数不等长问题。有关三角函数的距离测度和相似性测度也被广泛研究。例如直觉模糊集上的余弦相似性测度[10]打破了评价信息类型与维数限制,可以直观得出云服务质量的优劣。文献[11]将直觉模糊相似性测度拓展到区间q阶Orthopair模糊上,提出了正弦相似性测度。文献[12]提出的Hausdorff距离测度将欧几里得距离运用到2个单点集之间,定义了24种距离测度,解决了集合元素个数不等长的问题。本文受Hausdorff距离测度[12]的启发提出了基于正弦、余弦和正切的距离测度和加权距离测度,这些距离测度不需要犹豫模糊数所含元素个数等长;然后,通过讨论验证了上述测度具有有界性、对称性、三角不等式性等优良性质;最后,通过多属性决策的案例,将所提方法与现有的方法进行比较分析。结果显示,所提方法具有有效性和实用性。

1 预备知识

定义1[4] 给定任意非空有限集合X,则称

$$A=\left\{\left\langle x,h_A(x)\right\rangle : x\in X\right\}$$

为犹豫模糊集(HFS),其中,$h(x)\in[0,1]$表示$x\in X$属于X的子集A的所有可能程度,称$h_A(x)$为x关于A的犹豫模糊数(HFN)。

定义2[13] 对于犹豫模糊数h,其中l_h为h中所含元素的个数,其得分函数为

$$s(h)=\frac{1}{l_h}\sum_{\gamma\in h}\gamma。\tag{1}$$

定义3[14] 对于犹豫模糊数h,其中l_h为h中所含元素的个数,其方差函数为

$$v(h)=\frac{2}{l_h(l_h-1)}\sqrt{\sum_{\gamma_i,\gamma_j\in h}(\gamma_i-\gamma_j)^2}。\tag{2}$$

对于2个犹豫模糊数h_1、h_2比较大小,文献[15]给出如下法则:

(1)若$s(h_1)>s(h_2)$,则$h_1>h_2$;

(2)若$s(h_1)=s(h_2)$,则当$v(h_1)<v(h_2)$时,$h_1>h_2$;当$v(h_1)=v(h_2)$时,$h_1=h_2$。

文献[5]提出的广义犹豫加权Hamming距离测度如下所示:

$$d_{ghw}(A,B)=\left[\sum_{k=1}^{n}\omega_k\left(\frac{1}{l_{x_k}}\sum_{j=1}^{l_{x_k}}\left|h_A^{\sigma(i)}(x_k)h_B^{\sigma(j)}(x_k)\right|^{\lambda}\right)\right]^{1/\lambda}。\tag{3}$$

文献[6]考虑到每个犹豫模糊元素的犹豫程度,定义了如下公式:

$$d_{whug}(A,B)=\left[\frac{1}{2}\sum_{k=1}^{n}\omega_k\left(|u(h_A(x_k))-u(h_B(x_k))|^{\lambda}+\frac{1}{l_{x_k}}\sum_{j=1}^{l_{x_k}}|h_A^{\sigma(j)}(x_k)-h_B^{\sigma(j)}(x_k)|^{\lambda}\right)\right]^{1/\lambda},\tag{4}$$

其中,记$h_A^{\sigma(i)}(x_k)$为$h_A(x_k)$中第i小的元素,$h_B^{\sigma(i)}$为h_B中第j小的元素,$l_{h_A(x_k)}$、$l_{h_B(x_k)}$为犹豫模糊数$h_A(x_k)$、$h_B(x_k)$所含元素的个数。$l_{x_k}=\max(l_{h_A(x_k)},l_{h_B(x_k)})$;$\omega_k(k=1,2,\cdots,n)$为元素$x_k$的相关权重,满足$\omega_k\in[0,1]$,且$\sum_{k=1}^{n}\omega_k=1$;$u(h_A(x_k))=1-\frac{1}{l_{h_A(x_k)}}$,$u(h_B(x_k))=1-\frac{1}{l_{h_B(x_k)}}$,$\lambda\geqslant 1$。

一般情况下,可以用欧几里得距离$\|\cdot\|$来表示单个点a和集合B之间的距离θ,其数学形式为$\theta(a,B)=\min_{b\in B}\|a-b\|$。文献[16]将其扩展到犹豫模糊数上,提出犹豫模糊数之间的距离测度。

定义4[16] 假设对任意$x_k\in X=\{x_1,x_2,\cdots,x_n\}$,$h_A(x_k)=\left\{\left\langle x_k,\{h_A^{\sigma(i)}(x_k)|i=1,2,\cdots,l_{h_{A(x_k)}}\}\right\rangle : x_k\in X\right\}$,$h_B(x_k)=\left\{\left\langle x_k,\{h_B^{\sigma(j)}(x_k)|j=1,2,\cdots,l_{h_{B(x_k)}}\}\right\rangle : x_k\in X\right\}$为2个犹豫模糊数,可定义$h_A(x_k)$和$h_B(x_k)$之间的距离如下所示:

$$f_1(h_A(x_k),h_B(x_k)) = \max\{d_1(h_A(x_k),h_B(x_k)),d_1(h_B(x_k),h_A(x_k))\} = \max\left\{\max_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|,\max_{h_B^{\sigma(j)}\in h_B}\min_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\right\}, \tag{5}$$

$$f_2(h_A(x_k),h_B(x_k)) = \frac{1}{l_{h_{A(x_k)}}+l_{h_{B(x_k)}}}(d_2(h_A(x_k),h_B(x_k))+d_2(h_B(x_k),h_A(x_k))) = \frac{1}{l_{h_{A(x_k)}}+l_{h_{B(x_k)}}}\left(\sum_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|+\sum_{h_B^{\sigma(j)}\in h_B}\min_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\right), \tag{6}$$

其中

$$d_1(h_A(x_k),h_B(x_k)) = \max_{h_A^{\sigma(i)}\in h_A}\theta(h_A^{\sigma(i)}(x_k),h_B(x_k)) = \max_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|, \tag{7}$$

$$d_1(h_B(x_k),h_A(x_k)) = \max_{h_B^{\sigma(j)}\in h_B}\theta(h_B^{\sigma(j)}(x_k),h_A(x_k)) = \max_{h_B^{\sigma(j)}\in h_B}\min_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|, \tag{8}$$

$$d_2(h_A(x),h_B(x)) = \sum_{h_A^{\sigma(i)}\in h_A}\theta(h_A^{\sigma(i)}(x),h_B(x)) = \sum_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x)-h_B^{\sigma(j)}(x)\|, \tag{9}$$

$$d_2(h_B(x),h_A(x)) = \sum_{h_B^{\sigma(j)}\in h_B}\theta(h_B^{\sigma(j)}(x),h_A(x)) = \sum_{h_B^{\sigma(j)}\in h_B}\min_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x)-h_B^{\sigma(j)}(x)\|, \tag{10}$$

$\theta(h_A^{\sigma(i)}(x_k),h_B(x_k)) = \min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|$，表示犹豫模糊数$h_A(x_k)$中某一犹豫模糊元素$h_A^{\sigma(i)}(x_k)$到另一犹豫模糊数$h_B(x_k)$的最小欧式距离。

2 基于三角函数的新距离测度

定义4是基于Hausdorff距离测度的基础上定义的，不受元素个数等长的限制，本节结合三角函数提出犹豫模糊集的新距离测度、加权距离测度和含偏好的加权距离测度，并对其性质进行讨论。

定义5　给定任意非空有限集合$X=\{x_k|k=1,2,\cdots,n\}$，设$A=\{\langle x_k,h_A(x_k)\rangle:x_k\in X\}=\left\{\left\langle x_k,\left\{h_A^{\sigma(i)}(x_k)|i=1,2,\cdots,l_{h_{A(x_k)}}\right\}\right\rangle:x_k\in X\right\}$和$B=\{\langle x_k,h_B(x_k)\rangle:x_k\in X\}=\left\{\left\langle x_k,\left\{h_B^{\sigma(j)}(x_k)|j=1,2,\cdots,l_{h_{B(x_k)}}\right\}\right\rangle:x_k\in X\right\}$是$X$上的任意2个犹豫模糊集，结合式(5)和式(6)，分别定义A和B的距离测度如下所示：

$$SD_1(A,B) = \frac{1}{n}\sum_{k=1}^{n}\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_B(x_k)))\right] = \frac{1}{n}\sum_{k=1}^{n}\sin\left[\frac{\pi}{2}\left(\max\left\{\max_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|,\max_{h_B^{\sigma(j)}\in h_B}\min_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\right\}\right)\right], \tag{11}$$

$$SD_2(A,B) = \frac{1}{n}\sum_{k=1}^{n}\sin\left[\frac{\pi}{2}(f_2(h_A(x_k),h_B(x_k)))\right] = \frac{1}{n}\sum_{k=1}^{n}\sin\left[\frac{\pi}{2}\left(\frac{1}{l_{h_{A(x_k)}}+l_{h_{B(x_k)}}}\left(\sum_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|+\sum_{h_B^{\sigma(j)}\in h_B}\min_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\right)\right)\right], \tag{12}$$

$$CD_1(A,B) = 1-\frac{1}{n}\sum_{k=1}^{n}\cos\left[\frac{\pi}{2}(f_1(h_A(x_k),h_B(x_k)))\right] = 1-\frac{1}{n}\sum_{k=1}^{n}\cos\left[\frac{\pi}{2}\left(\max\left\{\max_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|,\max_{h_B^{\sigma(j)}\in h_B}\min_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\right\}\right)\right], \tag{13}$$

$$CD_2(A,B) = 1-\frac{1}{n}\sum_{k=1}^{n}\cos\left[\frac{\pi}{2}(f_2(h_A(x_k),h_B(x_k)))\right] = 1-\frac{1}{n}\sum_{k=1}^{n}\cos\left[\frac{\pi}{2}\left(\frac{1}{l_{h_{A(x_k)}}+l_{h_{B(x_k)}}}\left(\sum_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|+\sum_{h_B^{\sigma(j)}\in h_B}\min_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\right)\right)\right], \tag{14}$$

$$TD_1(A,B)=\frac{1}{n}\sum_{k=1}^{n}\tan\left[\frac{\pi}{4}\left(f_1(h_A(x_k),h_B(x_k)\right)\right]=$$

$$\frac{1}{n}\sum_{k=1}^{n}\tan\left[\frac{\pi}{4}\left(\max\left\{\max_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|,\max_{h_B^{\sigma(j)}\in h_B}\min_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\right\}\right)\right], \tag{15}$$

$$TD_2(A,B)=\frac{1}{n}\sum_{k=1}^{n}\tan\left[\frac{\pi}{4}\left(f_2(h_A(x_k),h_B(x_k)\right)\right]=$$

$$\frac{1}{n}\sum_{k=1}^{n}\tan\left[\frac{\pi}{4}\left(\frac{1}{l_{h_{A(x_k)}}+l_{h_{B(x_k)}}}\left(\sum_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|+\sum_{h_B^{\sigma(j)}\in h_B}\min_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\right)\right)\right]。 \tag{16}$$

定理1 给定任意非空有限集合$X=\{x_k|k=1,2,\cdots,n\}$,设A、B和C是X上的任意3个犹豫模糊集,$SD_g(g=1,2)$满足以下性质:

(S1)$0\leqslant SD_g(A,B)\leqslant 1$;(有界性)

(S2)$SD_g(A,B)=0$,当且仅当$A=B$;(条件反身性)

(S3)$SD_g(A,B)=SD_g(B,A)$;(对称性)

(S4)若$A<B<C$,即对任意$x_k\in X$有$h_A^{\sigma(i)}(x_k)\leqslant h_B^{\sigma(j)}(x_k)\leqslant h_C^{\sigma(p)}(x_k)$,则$SD_g(A,B)\leqslant SD_g(A,C)$且$SD_g(B,C)\leqslant SD_g(A,C)$。(包含性)

(S5)$SD_1(A,C)\leqslant SD_1(A,B)+SD_1(B,C)$。(三角不等式性)

证明 当$g=1$时,

(S1)由于$h_A(x_k),h_B(x_k)\in[0,1]$,根据式(7)、式(8)可知,$d_1(h_A(x_k),h_B(x_k))\in[0,1]$,$d_1(h_B(x_k),h_A(x_k))\in[0,1]$,则由式(5)可得$f_1(h_A(x_k),h_B(x_k))\in[0,1]$,可推出$\sin\left[\frac{\pi}{2}\left(f_1(h_A(x_k),h_B(x_k))\right)\right]\in[0,1]$,则$SD_1(A,B)\in[0,1]$,故有$0\leqslant SD_1(A,B)\leqslant 1$成立。

(S2)若$SD_1(A,B)=0$,则$SD_1(A,B)=\frac{1}{n}\sum_{k=1}^{n}\sin\left[\frac{\pi}{2}\left(f_1(h_A(x_k),h_B(x_k))\right)\right]=0$,当且仅当对任意$x_k\in X$,$f_1(h_A(x_k),h_B(x_k))=0$,可得$d_1(h_A(x_k),h_B(x_k))=0$,$d_1(h_B(x_k),h_A(x_k))=0$,即$\min_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|=0$,$\max_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|=0$且$\min_{h_B^{\sigma(j)}\in h_B}\min_{h_A^{\sigma(i)}\in h_A}\left\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\right\|=0$,$\max_{h_B^{\sigma(j)}\in h_B}\min_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|=0$。因此,对任意$h_A^{\sigma(i)}(x_k)\in A$,$h_B^{\sigma(i)}(x_k)\in B$有$\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|=0$,从而$h_A^{\sigma(i)}(x_k)=h_B^{\sigma(j)}(x_k)$,$h_A(x_k)=h_B(x_k)$,即$A=B$。

若$A=B$,则对任意$x_k\in X$,$\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|=0$,有$d_1(h_A(x_k),h_B(x_k))=0$,$d_1(h_B(x_k),h_A(x_k))=0$,即$f_1(h_A(x_k),h_B(x_k))=0$,从而$SD_1(A,B)=\frac{1}{n}\sum_{k=1}^{n}\sin\left[\frac{\pi}{2}\left(f_1(h_A(x_k),h_B(x_k))\right)\right]=0$。因此$SD_1(A,B)=0$,当且仅当$A=B$。

(S3)显然。

(S4)设$A<B<C$,对任意$x_k\in X$,$h_A^{\sigma(i)}(x_k)\in A$,$h_B^{\sigma(j)}(x_k)\in B$,$h_C^{\sigma(p)}(x_k)\in C$,有$h_A^{\sigma(i)}(x_k)\leqslant h_B^{\sigma(j)}(x_k)\leqslant h_C^{\sigma(p)}(x_k)$。则$\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\leqslant\|h_A^{\sigma(i)}(x_k)-h_C^{\sigma(p)}(x_k)\|$,根据式(7)可得$\min_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\leqslant\min_{h_A^{\sigma(p)}\in h_A}\min_{h_C^{\sigma(j)}\in h_C}\|h_A^{\sigma(i)}(x_k)-h_C^{\sigma(p)}(x_k)\|$,$\max_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\leqslant\max_{h_A^{\sigma(p)}\in h_A}\min_{h_C^{\sigma(j)}\in h_C}\|h_A^{\sigma(i)}(x_k)-h_C^{\sigma(p)}(x_k)\|$,可推出$d_1(h_A(x_k),h_B(x_k))\leqslant d_1(h_A(x_k),h_C(x_k))$,同理$d_1(h_B(x_k),h_A(x_k))\leqslant d_1(h_C(x_k),h_A(x_k))$,结合式(5)有$f_1(h_A(x_k),h_B(x_k))=\max\{d_1(h_A(x_k),h_B(x_k)),d_1(h_B(x_k),h_A(x_k))\}\leqslant\max\{d_1(h_A(x_k),h_C(x_k)),d_1(h_C(x_k),h_A(x_k))\}$。即$f_1(h_A(x_k),h_B(x_k))\leqslant f_1(h_A(x_k),h_C(x_k))$,由式(11)可推出$\frac{1}{n}\sum_{k=1}^{n}\sin\left[\frac{\pi}{2}\left(f_1(h_A(x_k),h_B(x_k))\right)\right]\leqslant\frac{1}{n}\sum_{k=1}^{n}\sin\left[\frac{\pi}{2}\left(f_1(h_A(x_k),h_C(x_k))\right)\right]$,因此$SD_1(A,B)\leqslant SD_1(A,C)$,同理可得$SD_1(B,C)\leqslant SD_1(A,C)$。

(S5)由文献[16]已经证明$f_1(h_A(x_k),h_C(x_k))\leqslant f_1(h_A(x_k),h_B(x_k))+f_1(h_B(x_k),h_C(x_k))$,这里不再说明。下面

对$f_1(h_A(x_k),h_C(x_k)),f_1(h_A(x_k),h_B(x_k)),f_1(h_B(x_k),h_C(x_k))$在区间$[0,1]$上的取值进行分类讨论。

(1)当$f_1(h_A(x_k),h_C(x_k))\in[0,1],f_1(h_A(x_k),h_B(x_k)),f_1(h_B(x_k),h_C(x_k))\in[0,0.5]$时,可推出$0\leqslant f_1(h_A(x_k),h_B(x_k))+f_1(h_B(x_k),h_C(x_k))\leqslant 1$,又因为$f_1(h_A(x_k),h_C(x_k))\leqslant f_1(h_A(x_k),h_B(x_k))+f_1(h_B(x_k),h_C(x_k))$,则

$$\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_C(x_k)))\right]\leqslant\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_B(x_k)+f_1(h_B(x_k),h_C(x_k))\right]\leqslant$$

$$\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_B(x_k)))\right]+\sin\left[\frac{\pi}{2}(f_1(h_B(x_k),h_C(x_k)))\right]。$$

(2)当$f_1(h_A(x_k),h_C(x_k))\in[0,0.5],f_1(h_A(x_k),h_B(x_k))\in[0,0.5],f_1(h_B(x_k),h_C(x_k))\in(0.5,1]$时,可得$0\leqslant\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_C(x_k)))\right]\leqslant\frac{\sqrt{2}}{2},0\leqslant\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_B(x_k)))\right]\leqslant\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}<\sin\left[\frac{\pi}{2}(f_1(h_B(x_k),h_C(x_k)))\right]\leqslant 1$,

则
$$\frac{\sqrt{2}}{2}\leqslant\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_B(x_k)))\right]+\sin\left[\frac{\pi}{2}(f_1(h_B(x_k),h_C(x_k)))\right]\leqslant 1+\frac{\sqrt{2}}{2},$$

故
$$\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_C(x_k)))\right]\leqslant\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_B(x_k)))\right]+\sin\left[\frac{\pi}{2}(f_1(h_B(x_k),h_C(x_k)))\right]。$$

当$f_1(h_A(x_k),h_C(x_k))\in[0.5,1],f_1(h_A(x_k),h_B(x_k))\in[0,0.5],f_1(h_B(x_k),h_C(x_k))\in(0.5,1]$时,

情形1:若$f_1(h_A(x_k),h_C(x_k))\leqslant\max\{f_1(h_A(x_k),h_B(x_k)),f_1(h_B(x_k),h_C(x_k))\}$,则

$$\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_C(x_k)))\right]\leqslant\max\left\{\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_B(x_k)))\right],\sin\left[\frac{\pi}{2}(f_1(h_B(x_k),h_C(x_k)))\right]\right\},$$

可推出
$$\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_C(x_k)))\right]\leqslant\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_B(x_k)))\right]+\sin\left[\frac{\pi}{2}(f_1(h_B(x_k),h_C(x_k)))\right]。$$

情形2:若$f_1(h_A(x_k),h_C(x_k))\geqslant f_1(h_B(x_k),h_C(x_k))$,令$f_1(h_A(x_k),h_C(x_k))=f_1(h_B(x_k),h_C(x_k))+t(0\leqslant t\leqslant 0.5)$,又因为$f_1(h_A(x_k),h_C(x_k))\leqslant f_1(h_A(x_k),h_B(x_k))+f_1(h_B(x_k),h_C(x_k))$,则$t\leqslant f_1(h_A(x_k),h_B(x_k))$,

$$\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_C(x_k)))\right]=\sin\left[\frac{\pi}{2}\left(f_1(h_B(x_k),h_C(x_k))+t\right)\right]\leqslant\sin\left[\frac{\pi}{2}\left(f_1(h_B(x_k),h_C(x_k))\right)\right]+\sin\left[\frac{\pi}{2}(t)\right]\leqslant$$

$$\sin\left[\frac{\pi}{2}\left(f_1(h_B(x_k),h_C(x_k))\right)\right]+\sin\left[\frac{\pi}{2}\left(f_1(h_A(x_k),h_B(x_k))\right)\right]。$$

(3)当$f_1(h_A(x_k),h_C(x_k))\in[0,1],f_1(h_A(x_k),h_B(x_k))\in(0.5,1],f_1(h_B(x_k),h_C(x_k))\in[0,0.5]$时,同理可得$\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_C(x_k)))\right]\leqslant\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_B(x_k)))\right]+\sin\left[\frac{\pi}{2}(f_1(h_B(x_k),h_C(x_k)))\right]$。

(4)当$f_1(h_A(x_k),h_B(x_k)),f_1(h_B(x_k),h_C(x_k))\in(0.5,1]$时,$\frac{\sqrt{2}}{2}<\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_B(x_k)))\right]\leqslant 1$,$\frac{\sqrt{2}}{2}<\sin\left[\frac{\pi}{2}(f_1(h_B(x_k),h_C(x_k)))\right]\leqslant 1$,可得$\sqrt{2}<\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_B(x_k)))\right]+\sin\left[\frac{\pi}{2}(f_1(h_B(x_k),h_C(x_k)))\right]\leqslant 2$,又因为$f_1(h_A(x_k),h_C(x_k))\in[0,1]$,则$0\leqslant\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_C(x_k)))\right]\leqslant 1$,可推出$\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_C(x_k)))\right]<\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_B(x_k)))\right]+\sin\left[\frac{\pi}{2}(f_1(h_B(x_k),h_C(x_k)))\right]$。

综上,对任意$x_k\in X$,有

$\frac{1}{n}\sum_{k=1}^{n}\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_C(x_k)))\right]\leqslant\frac{1}{n}\sum_{k=1}^{n}\sin\left[\frac{\pi}{2}(f_1(h_A(x_k),h_B(x_k)))\right]+\frac{1}{n}\sum_{k=1}^{n}\sin\left[\frac{\pi}{2}(f_1(h_B(x_k),h_C(x_k)))\right]$,则$SD_1(A,C)\leqslant SD_1(A,B)+SD_1(B,C)$成立。

当$g=2$时,(S1)~(S4)证明与上述类似,这里不再说明。

备注1 SD_2不满足三角不等式性。

例1 假设犹豫模糊集$A=\{\langle x_1,0.1\rangle\},B=\{\langle x_1,0.1,0.2,0.9\rangle\},C=\{\langle x_1,0.9\rangle\}$。利用式(12)分别计算三者

两两之间的SD_2值为

$$SD_2(A,B)=\sin\frac{\pi}{2}\left(\frac{1}{1+3}(0+0+0.1+0.8)\right)=0.346\,1,$$

$$SD_2(B,C)=\sin\frac{\pi}{2}\left(\frac{1}{3+1}(0.8+0.7+0+0)\right)=0.555\,6,$$

$$SD_2(A,C)=\sin\frac{\pi}{2}\left(\frac{1}{1+1}(0.8+0.8)\right)=0.951\,1,$$

则$SD_2(A,B)+SD_2(B,C)=0.901\ 7<SD_2(A,C)=0.951\ 1$。

故SD_2不满足三角不等式性质。

备注2 CD_g、$TD_g(g=1,2)$满足定理1中的性质(S1)~(S4)。

备注3 CD_g、$TD_g(g=1,2)$不满足三角不等式性。

例2 假设犹豫模糊集$A=\{\langle x_1,0.1\rangle\},B=\{\langle x_1,0.5\rangle\},C=\{\langle x_1,0.1,0.2,0.9\rangle\}$。利用式(13)、式(15)分别计算三者两两之间的CD_1、TD_1值为

$$CD_1(A,B)=1-\cos\frac{\pi}{2}(0.4)=0.191\,0,$$

$$CD_1(B,C)=1-\cos\frac{\pi}{2}(0.4)=0.191\,0,$$

$$CD_1(A,C)=1-\cos\frac{\pi}{2}(0.8)=0.691\,0,$$

则$CD_1(A,B)+CD_1(B,C)=0.382\,0<CD_1(A,C)=0.691\,0$。

$$TD_1(A,B)=\tan\frac{\pi}{4}(0.4)=0.324\,9,$$

$$TD_1(B,C)=\tan\frac{\pi}{4}(0.4)=0.324\,9,$$

$$TD_1(A,C)=\tan\frac{\pi}{4}(0.8)=0.726\,5,$$

则$TD_1(A,B)+TD_1(B,C)=0.649\,8<TD_1(A,C)=0.726\,5$。

故CD_1、TD_1不满足三角不等式性质。

例3 我们使用例1中的数据，利用式(14)、式(16)分别计算三者两两之间的CD_2、TD_2值为

$$CD_2(A,B)=1-\cos\frac{\pi}{2}\left(\frac{1}{1+3}(0+0+0.1+0.8)\right)=0.061\,8,$$

$$CD_2(B,C)=1-\cos\frac{\pi}{2}\left(\frac{1}{3+1}(0.8+0.7+0+0)\right)=0.168\,5,$$

$$CD_2(A,C)=1-\cos\frac{\pi}{2}\left(\frac{1}{1+1}(0.8+0.8)\right)=0.691\,0,$$

则$CD_2(A,B)+CD_2(B,C)=0.230\,3<CD_2(A,C)=0.691\ 0$。

$$TD_2(A,B)=\tan\frac{\pi}{4}\left(\frac{1}{1+3}(0+0+0.1+0.8)\right)=0.178\,6,$$

$$TD_2(B,C)=\tan\frac{\pi}{4}\left(\frac{1}{3+1}(0.8+0.7+0+0)\right)=0.303\,3,$$

$$TD_2(A,C)=\tan\frac{\pi}{4}\left(\frac{1}{1+1}(0.8+0.8)\right)=0.726\,5,$$

则$TD_2(A,B)+TD_2(B,C)=0.481\ 9<TD_2(A,C)=0.726\,5$。

故CD_2、TD_2不满足三角不等式性质。

定义6 给定任意非空有限集合$X=\{x_k|k=1,2,\cdots,n\}$，设A，B和C是X上的任意3个犹豫模糊集，对任意$x_k\in X$，考虑每个元素x_k的权重。假设元素x_k的权重为$\omega_k(k=1,2,\cdots,n)$，满足$\omega_k\in[0,1]$，且$\sum_{k=1}^{n}\omega_k=1$。可以在犹豫模糊集A和B之间引入以下加权测度：

$$WSD_1(A,B)=\sum_{k=1}^{n}\omega_k \sin\left[\frac{\pi}{2}\left(f_1(h_A(x_k),h_B(x_k)\right)\right]= \\ \sum_{k=1}^{n}\omega_k \sin\left[\frac{\pi}{2}\left(\max\left\{\max_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|,\max_{h_B^{\sigma(j)}\in h_B}\min_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\right\}\right)\right], \tag{17}$$

$$WSD_2(A,B)=\sum_{k=1}^{n}\omega_k \sin\left[\frac{\pi}{2}\left(f_2(h_A(x_k),h_B(x_k)\right)\right]= \\ \sum_{k=1}^{n}\omega_k \sin\left[\frac{\pi}{2}\left(\frac{1}{l_{h_{A(x_k)}}+l_{h_{B(x_k)}}}\left(\sum_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|+\sum_{h_B^{\sigma(j)}\in h_B}\min_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\right)\right)\right], \tag{18}$$

$$WCD_1(A,B)=1-\sum_{k=1}^{n}\omega_k \cos\left[\frac{\pi}{2}\left(f_1(h_A(x_k),h_B(x_k)\right)\right]= \\ 1-\sum_{k=1}^{n}\omega_k \cos\left[\frac{\pi}{2}\left(\max\left\{\max_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|,\max_{h_B^{\sigma(j)}\in h_B}\min_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\right\}\right)\right], \tag{19}$$

$$WCD_2(A,B)=1-\sum_{k=1}^{n}\omega_k \cos\left[\frac{\pi}{2}\left(f_2(h_A(x_k),h_B(x_k)\right)\right]= \\ 1-\sum_{k=1}^{n}\omega_k \cos\left[\frac{\pi}{2}\left(\frac{1}{l_{h_{A(x_k)}}+l_{h_{B(x_k)}}}\left(\sum_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|+\sum_{h_B^{\sigma(j)}\in h_B}\min_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\right)\right)\right], \tag{20}$$

$$WTD_1(A,B)=\sum_{k=1}^{n}\omega_k \tan\left[\frac{\pi}{4}\left(f_1(h_A(x_k),h_B(x_k)\right)\right]= \\ \sum_{k=1}^{n}\omega_k \tan\left[\frac{\pi}{4}\left(\max\left\{\max_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|,\max_{h_B^{\sigma(j)}\in h_B}\min_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\right\}\right)\right], \tag{21}$$

$$WTD_2(A,B)=\sum_{k=1}^{n}\omega_k \tan\left[\frac{\pi}{4}\left(f_2(h_A(x_k),h_B(x_k)\right)\right]= \\ \sum_{k=1}^{n}\omega_k \tan\left[\frac{\pi}{4}\left(\frac{1}{l_{h_{A(x_k)}}+l_{h_{B(x_k)}}}\left(\sum_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|+\sum_{h_B^{\sigma(j)}\in h_B}\min_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\right)\right)\right]。 \tag{22}$$

特别地,当$\omega_k=\frac{1}{n}$时,式(17)~式(22)可分别退化为式(11)~式(16)。

如果考虑$f_1(h_A(x_k),h_B(x_k))$和$f_2(h_A(x_k),h_B(x_k))$的偏好影响,那么一些具有偏好的基于三角函数的距离测度定义如下:

定义7 给定任意非空有限集合$X=\{x_k|k=1,2,\cdots,n\}$,设A和B是X上的任意2个犹豫模糊集,α和β分别是$f_1(h_A(x_k),h_B(x_k))$和$f_2(h_A(x_k),h_B(x_k))$的权重,$f_1(h_A(x_k),h_B(x_k))$和$f_2(h_A(x_k),h_B(x_k))$分别由式(5)和式(6)给出,满足$\alpha+\beta=1$,且$0\leqslant\alpha,\beta\leqslant1$,则将$A$和$B$之间的偏好加权距离定义如下:

$$WPSD(A,B)=\sum_{k=1}^{n}\omega_k \sin\left[\frac{\pi}{2}\left(\alpha f_1(h_A(x_k),h_B(x_k)+\beta f_2(h_A(x_k),h_B(x_k)\right)\right]= \\ \sum_{k=1}^{n}\omega_k \sin\left[\frac{\pi}{2}\left(\begin{gathered}\alpha\left(\max\left\{\max_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|,\max_{h_B^{\sigma(j)}\in h_B}\min_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\right\}\right)+ \\ \beta\left(\frac{1}{l_{h_{A(x_k)}}+l_{h_{B(x_k)}}}\left(\sum_{h_A^{\sigma(i)}\in h_A}\min_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|+\sum_{h_B^{\sigma(j)}\in h_B}\min_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\right)\right)\end{gathered}\right)\right], \tag{23}$$

$$WPCD(A,B)=1-\sum_{k=1}^{n}\omega_k \cos\left[\frac{\pi}{2}\left(\alpha f_1(h_A(x_k),h_B(x_k)+\beta f_2(h_A(x_k),h_B(x_k)\right)\right]=$$

$$1-\sum_{k=1}^{n}\omega_k\cos\left[\frac{\pi}{2}\left(\begin{array}{l}\alpha\left(\max\left\{\max\limits_{h_A^{\sigma(i)}\in h_A}\min\limits_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|,\max\limits_{h_B^{\sigma(j)}\in h_B}\min\limits_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\right\}\right)+\\ \beta\left(\frac{1}{l_{h_{A(x_k)}}+l_{h_{B(x_k)}}}\left(\sum_{h_A^{\sigma(i)}\in h_A}\min\limits_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|+\sum_{h_B^{\sigma(j)}\in h_B}\min\limits_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\right)\right)\end{array}\right)\right], \tag{24}$$

$$WPTD(A,B)=\sum_{k=1}^{n}\omega_k\tan\left[\frac{\pi}{4}\left(\alpha f_1(h_A(x_k),h_B(x_k))+\beta f_2(h_A(x_k),h_B(x_k))\right)\right]=$$

$$\sum_{k=1}^{n}\omega_k\tan\left[\frac{\pi}{4}\left(\begin{array}{l}\alpha\left(\max\left\{\max\limits_{h_A^{\sigma(i)}\in h_A}\min\limits_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|,\max\limits_{h_B^{\sigma(j)}\in h_B}\min\limits_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\right\}\right)+\\ \beta\left(\frac{1}{l_{h_{A(x_k)}}+l_{h_{B(x_k)}}}\left(\sum_{h_A^{\sigma(i)}\in h_A}\min\limits_{h_B^{\sigma(j)}\in h_B}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|+\sum_{h_B^{\sigma(j)}\in h_B}\min\limits_{h_A^{\sigma(i)}\in h_A}\|h_A^{\sigma(i)}(x_k)-h_B^{\sigma(j)}(x_k)\|\right)\right)\end{array}\right)\right]。 \tag{25}$$

特别地，当$\alpha = 0$时，式(23)~ 式(25)将分别退化为式(18)、式(20)和式(22)；当$\beta = 0$时，式(23)~式(25)将分别退化为式(17)、式(19)和式(21)。

3 在多属性决策中的应用

本节将所提距离测度运用到2种理想解的多属性决策方法中，并与文献[5-6]所提方法进行了对比分析，来说明本文所提方法的有效性。

例4[5] 选择恰当的能源政策对社会经济和环境的影响尤为重要。假设有5种能源替代方案$A_i(i = 1,2,3,4,5)$，应考虑4个属性：C_1：技术；C_2：环境；C_3：社会政治；C_4：经济，邀请了多个决策者对5种备选方案进行评估，决策者评估的结果显示在犹豫模糊决策矩阵中，如表1所示，属性对应的权重向量是$\omega = (0.15,0.3,0.2,0.35)^{\mathrm{T}}$。

表1 犹豫模糊决策矩阵

Table 1 Hesitant fuzzy decision matrix

	C_1	C_2	C_3	C_4
A_1	$\{0.3,0.4,0.5\}$	$\{0.1,0.7,0.8,0.9\}$	$\{0.2,0.4,0.5\}$	$\{0.3,0.5,0.6,0.9\}$
A_2	$\{0.3,0.5\}$	$\{0.2,0.5,0.6,0.7,0.9\}$	$\{0.1,0.5,0.6,0.8\}$	$\{0.3,0.4,0.7\}$
A_3	$\{0.6,0.7\}$	$\{0.6,0.9\}$	$\{0.3,0.5,0.7\}$	$\{0.4,0.6\}$
A_4	$\{0.3,0.4,0.7,0.8\}$	$\{0.2,0.4,0.7\}$	$\{0.1,0.8\}$	$\{0.6,0.8,0.9\}$
A_5	$\{0.1,0.3,0.6,0.7,0.8\}$	$\{0.4,0.6,0.7,0.8\}$	$\{0.7,0.8,0.9\}$	$\{0.3,0.6,0.7,0.9\}$

方法1：这里假设特殊的理想方案为$A^* = \{1,1,1,1\}$，通过距离测度计算备选方案$A_i(i = 1,2,3,4,5)$与理想方案A^*两两之间的距离，距离越近，方案越优，计算结果见表2。

在表2中，d_{hgw}[5]的距离测度结果在很大程度上取决于参数λ的变化，也就是说，当$\lambda\in[1,3]$时，最优方案是A_5，当$\lambda\in(3,\infty)$时，最优方案变为A_3。而d_{whug}[6]并不受参数λ的影响，最优方案是A_3。利用WSD_1、WCD_1和WTD_1的最优方案都是A_5，利用WSD_2、WCD_2和WTD_2的最优方案都是A_3。

方法2：在例4的基础上，理想解利用犹豫模糊TOPSIS的方法更新。下面介绍犹豫模糊TOPSIS决策方法，步骤如下：

步骤1：构建犹豫模糊决策矩阵$H = (h_{ij})_{m\times n}$，h_{ij}由第i个方案$A_i(i = 1,2,\cdots,m)$在第j个属性$C_j(j = 1,2,\cdots,n)$中一组可能的评估值组成，评估值由决策者给出，此外，决策者确定不同属性下的相对权重$\omega_k(k = 1,2,\cdots,n)$，满足$\omega_k\in[0,1]$，且$\sum\limits_{k=1}^{n}\omega_k = 1$。

步骤2：确定正理想解$A^+ = \{h_1^+,h_2^+,\cdots,h_n^+\}$和负理想解$A^- = \{h_1^-,h_2^-,\cdots,h_n^-\}$，定义如下：

表2 5个能源方案的距离结果

Table 2 Distance results of the five energy projects

方法	距离	A_1	A_2	A_3	A_4	A_5	排序
d_{hgw}[5]	$\lambda = 1$	0.479 9	0.502 7	0.402 5	0.429 2	0.355 8	$A_5 > A_3 > A_4 > A_1 > A_2$
	$\lambda = 2$	0.537 8	0.545 1	0.436 6	0.505 2	0.412 9	$A_5 > A_3 > A_4 > A_1 > A_2$
	$\lambda = 6$	0.659 9	0.647 6	0.515 6	0.670 4	0.569 9	$A_3 > A_5 > A_2 > A_1 > A_4$
	$\lambda = 10$	0.721 3	0.704 6	0.560 7	0.737 3	0.653 7	$A_3 > A_5 > A_2 > A_1 > A_4$
d_{whug}[6]	$\lambda = 1$	0.622 3	0.612 3	0.497 4	0.580 0	0.557 5	$A_3 > A_5 > A_4 > A_2 > A_1$
	$\lambda = 2$	0.657 0	0.640 7	0.512 0	0.618 6	0.606 2	$A_3 > A_5 > A_4 > A_2 > A_1$
	$\lambda = 6$	0.713 6	0.697 9	0.552 3	0.695 2	0.686 2	$A_3 > A_5 > A_4 > A_2 > A_1$
	$\lambda = 10$	0.741 8	0.727 1	0.580 9	0.736 4	0.715 9	$A_3 > A_5 > A_2 > A_4 > A_1$
本文所提方法	WSD_1	0.611 7	0.645 0	0.531 8	0.534 3	0.466 9	$A_5 > A_3 > A_4 > A_1 > A_2$
	WCD_1	0.231 2	0.242 9	0.172 6	0.178 2	0.123 6	$A_5 > A_3 > A_4 > A_1 > A_2$
	WTD_1	0.353 4	0.369 4	0.295 9	0.299 3	0.250 5	$A_5 > A_3 > A_4 > A_1 > A_2$
	WSD_2	0.932 0	0.928 4	0.725 9	0.822 2	0.793 5	$A_3 > A_5 > A_4 > A_2 > A_1$
	WCD_2	0.664 3	0.649 0	0.339 4	0.524 8	0.463 1	$A_3 > A_5 > A_4 > A_2 > A_1$
	WTD_2	0.707 9	0.695 2	0.447 1	0.594 4	0.543 5	$A_3 > A_5 > A_4 > A_2 > A_1$

$$h_j^+ = \begin{cases} \max\limits_{i=1,2,\ldots,m} \{h_{ij}\}, \text{对收益性属性} C_j \\ \min\limits_{i=1,2,\ldots,m} \{h_{ij}\}, \text{对成本性属性} C_j \end{cases}, h_j^- = \begin{cases} \min\limits_{i=1,2,\ldots,m} \{h_{ij}\}, \text{对收益性属性} C_j \\ \max\limits_{i=1,2,\ldots,m} \{h_{ij}\}, \text{对成本性属性} C_j \end{cases}。$$

步骤3：根据加权距离测度计算每个备选方案$A_i(i = 1,2,\cdots,m)$与正理想解$A^+ = \{h_1^+,h_2^+,\cdots,h_n^+\}$之间的距离$d(A_i, A^+)$以及与负理想解$A^- = \{h_1^-,h_2^-,\cdots,h_n^-\}$之间的距离$d_g(A_i, A^-)$。

步骤4：根据式(26)计算各方案的贴近度$R_i^g(g = 1,2)$：

$$R_i^g = \frac{d_i^{g-}}{d_i^{g-} + d_i^{g+}}。 \tag{26}$$

步骤5：通过贴近度的大小，得到方案$A_i(i = 1,2,\cdots,m)$的排序，贴近度越大，方案越优。

根据上述犹豫模糊TOPSIS决策算法步骤，计算结果如下：

步骤1：例4中的表1已给出犹豫模糊决策矩阵和属性权重。

步骤2：由于属性都是收益型属性，根据犹豫模糊数的得分函数式(1)和方差函数式(2)，可得正理想解$A^+ = \{h_1^+,h_2^+,\cdots,h_n^+\}$与负理想解$A^- = \{h_1^-,h_2^-,\cdots,h_n^-\}$：

$$A^+ = \{\langle 0.6,0.7\rangle,\langle 0.6,0.9\rangle,\langle 0.7,0.8,0.9\rangle,\langle 0.6,0.8,0.9\rangle\},$$
$$A^- = \{\langle 0.3,0.5\rangle,\langle 0.2,0.4,0.7\rangle,\langle 0.2,0.4,0.5\rangle,\langle 0.3,0.4,0.7\rangle\}。$$

步骤3：根据式(17)、式(18)，可得$WSD_g(A_i,A^+)$，$WSD_g(A_i,A^-)(g = 1,2)$结果见表3。

步骤4：由式(26)计算每个备选方案A_i与理想解的接近度$R_i^g(g = 1,2)$:

$$R_1^1 = 0.290\,3, R_2^1 = 0.216\,8, R_3^1 = 0.646\,0, R_4^1 = 0.438\,8, R_5^1 = 0.479\,7,$$
$$R_1^2 = 0.404\,5, R_2^2 = 0.219\,3, R_3^2 = 0.581\,4, R_4^2 = 0.498\,2, R_5^2 = 0.623\,7。$$

表3 方案A_i与A^+,A^-之间的距离

Table 3 The distance between project A_i and A^+, A^-

方案	$WSD_1(A_i,A^+)$	$WSD_1(A_i,A^-)$	$WSD_2(A_i,A^+)$	$WSD_2(A_i,A^-)$
A_1	0.580 5	0.267 8	0.238 8	0.111 9
A_2	0.565 1	0.183 5	0.220 4	0.064 9
A_3	0.276 5	0.361 0	0.123 0	0.192 5
A_4	0.406 2	0.317 8	0.175 8	0.185 7
A_5	0.357 7	0.430 5	0.111 7	0.192 5

步骤5:根据贴近度R_i^1的值对备选方案A_i进行排序,结果为$A_5 > A_3 > A_4 > A_1 > A_2$,最优方案是$A_5$。根据贴近度$R_i^2$的值对备选方案$A_i$进行排序,结果为$A_5 > A_3 > A_4 > A_1 > A_2$,最优方案也是$A_5$。

利用本文所提方法与文献[5-6]提出方法计算结果见表4。

表4 贴近度计算结果

Table 4 The calculation result of the relative closeness

方法	距离	A_1	A_2	A_3	A_4	A_5	排序
d_{hgw}[5]	$\lambda=1$	0.288 6	0.333 8	0.503 8	0.476 5	0.738 1	$A_5>A_3>A_4>A_2>A_1$
	$\lambda=2$	0.352 4	0.400 5	0.464 7	0.445 8	0.660 6	$A_5>A_3>A_4>A_2>A_1$
	$\lambda=6$	0.432 2	0.415 3	0.474 6	0.393 4	0.557 8	$A_5>A_3>A_1>A_2>A_4$
	$\lambda=10$	0.459 3	0.407 4	0.484 6	0.380 9	0.528 0	$A_5>A_3>A_1>A_2>A_4$
d_{whug}[6]	$\lambda=1$	0.384 2	0.354 2	0.529 0	0.415 2	0.611 3	$A_5>A_3>A_4>A_1>A_2$
	$\lambda=2$	0.416 2	0.403 0	0.476 0	0.430 2	0.590 5	$A_5>A_3>A_4>A_1>A_2$
	$\lambda=6$	0.471 9	0.415 3	0.474 9	0.393 4	0.552 8	$A_5>A_3>A_1>A_2>A_4$
	$\lambda=10$	0.484 1	0.407 4	0.484 6	0.380 9	0.527 7	$A_5>A_3>A_1>A_2>A_4$
本文所提方法	WSD_1	0.315 7	0.245 1	0.566 3	0.438 9	0.562 1	$A_5>A_3>A_4>A_1>A_2$
	WTD_1	0.298 6	0.230 2	0.563 9	0.416 5	0.569 7	$A_5>A_3>A_4>A_1>A_2$
	WCD_1	0.204 6	0.158 1	0.534 7	0.328 3	0.606 8	$A_5>A_3>A_4>A_1>A_2$
	WSD_2	0.319 0	0.227 4	0.610 0	0.513 6	0.632 8	$A_5>A_3>A_4>A_1>A_2$
	WCD_2	0.193 4	0.141 7	0.628 2	0.511 0	0.789 4	$A_5>A_3>A_4>A_1>A_2$
	WTD_2	0.313 6	0.225 6	0.611 0	0.513 6	0.639 5	$A_5>A_3>A_4>A_1>A_2$
$\alpha=0.5,\beta=0.5$	$WPSD$	0.313 4	0.236 8	0.579 6	0.458 1	0.582 2	$A_5>A_3>A_4>A_1>A_2$
	$WPTD$	0.304 5	0.230 9	0.579 6	0.450 2	0.589 9	$A_5>A_3>A_4>A_1>A_2$
	$WPCD$	0.205 8	0.156 3	0.564 8	0.382 2	0.668 8	$A_5>A_3>A_4>A_2>A_1$

由表4的结果可知,尽管排序结果并不完全一致,但通常认为最优能源政策是A_5。d_{hgw}[5]、d_{whug}[6]的距离测度结果在很大程度上取决于参数λ的变化,也就是说,当$\lambda \in [1,3]$时,排序结果为$A_5 > A_3 > A_4 > A_2 > A_1$,当$\lambda \in (3,\infty)$时,排序结果变为$A_5 > A_3 > A_1 > A_2 > A_4$。而本文所提方法的排序结果相同,最优方案都是$A_5$。原因在于文献[5-6]所提方法需要限制犹豫模糊数的隶属度个数等长,而本文所提方法不受这样的限制。值得说明的是,本文所提方法不依赖于d_{hgw}[5]和d_{whug}[6]中的参数λ,也能达到很好的效果,具有较好的有效性和实用性。

4 结论

本文提出的基于三角函数的犹豫模糊集距离测度是犹豫模糊集距离测度的拓展形式,具有满足测度的优良性质。将该方法运用到多属性决策的例子中,与现有的一些距离测度进行了比较分析,结果表明本文所提方法能表达较全面的数据信息,具有较好的实用性和有效性,为解决复杂犹豫模糊信息下的决策问题提供了有利工具。

参考文献:

[1] ZADEH L A. Fuzzy sets[J]. Information and Control,1965,8(3):338-353.
[2] ATANASSOV K T. Intuitionistic fuzzy sets[J]. Fuzzy Sets and Systems,1986,20(1):87-96.
[3] ATANASSOV K,GARGOV G. Interval valued intuitionistic fuzzy sets[J]. Fuzzy Sets and Systems,1989,31(3):343-349.
[4] TORRA V. Hesitant fuzzy sets[J]. International Journal of Intelligent Systems,2010,25(6):529-539.
[5] XU Z S,XIA M M. Distance and similarity measures for hesitant fuzzy sets[J]. Information Sciences,2011,181(11):2128-2138.
[6] LI D Q,ZENG W Y,ZHAO Y B. Note on distance measure of hesitant fuzzy sets[J]. Information Sciences,2015,321:103-115.

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matical Problems in Engineering,2017,2017:6549791.

[18] TANG J,MENG F Y. Ranking objects from group decision making with interval-valued hesitant fuzzy preference relations in view of additive consistency and consensus[J]. Knowledge-Based Systems,2018,162:46-61.

[19] ZHANG Y N,TANG J,MENG F Y. Programming model-based method for ranking objects from group decision making with interval-valued hesitant fuzzy preference relations[J]. Applied Intelligence,2019,49(3):837-857.

[20] TANG J,MENG F Y. New method for interval-valued hesitant fuzzy decision making based on preference relations[J]. Soft Computing,2020,24(17):13381-13399.

[21] TANG J,ZHANG Y N,FUJITA H,et al. Analysis of acceptable additive consistency and consensus of group decision making with interval-valued hesitant fuzzy preference relations[J]. Neural Computing and Applications,2021,33(13):7747-7772.

[22] 徐泽水,任珮嘉. 犹豫模糊偏好决策研究进展与前景[J]. 系统工程理论与实践,2020,40(8):2193-2202.

[23] XU Z S,DA Q L. The uncertain OWA operator[J]. International Journal of Intelligent Systems,2002,17(6):569-575.

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[7] 沙秀艳,尹传存. 基于隶属度偏差加权的改进犹豫模糊距离测度及应用[J]. 统计与决策,2022,38(4):180-184.

[8] ZHANG H M. Distance and entropy measures for dual hesitant fuzzy sets[J]. Computational and Applied Mathematics,2020,39(2):91.

[9] 李双明,关欣,孙贵东. 基于犹豫模糊集的不等长序列识别方法及应用[J]. 通信学报,2021,42(7):41-51.

[10] 王婕,耿秀丽. 基于直觉模糊余弦相似度的云制造服务质量评价[J]. 计算机集成制造系统,2021,27(4):1128-1134.

[11] 徐露,林志超,周礼刚,等. 基于区间q-Rung Orthopair模糊正弦相似测度的决策方法[J]. 武汉理工大学学报(信息与管理工程版),2022,44(3):69-74,111.

[12] DUBUISSON M P,JAIN A K. A modified Hausdorff distance for object matching[C]//Proceedings of 12th International Conference on Pattern Recognition. Jerusalem:IEEE,2002:566-568.

[13] XIA M M,XU Z S. Hesitant fuzzy information aggregation in decision making[J]. International Journal of Approximate Reasoning,2011,52(3):395-407.

[14] LIAO H C,XU Z S. Extended hesitant fuzzy hybrid weighted aggregation operators and their application in decision making[J]. Soft Computing,2015,19(9):2551-2564.

[15] LIAO H C,XU Z S,XIA M M. Multiplicative consistency of hesitant fuzzy preference relation and its application in group decision making[J]. International Journal of Information Technology & Decision Making,2014,13(1):47-76.

[16] FARHADINIA B,XU Z S. A novel distance-based multiple attribute decision-making with hesitant fuzzy sets[J]. Soft Computing,2020,24(7):5005-5017.

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